On the concept of quasi-truth

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Summary: In this paper we present a formal definition of pragmatic truth, the definition of *quasi-truth*: this concept, first introduced by da Costa and collaborators on trying to capture the meaning of theories of pragmatist thinkers such as Peirce and James, is considered as the truth conception inherent to empirical theories and a generalization (for partial contexts) of Tarski's correspondence characterization of truth. By using a semantical approach, we present a suitable logic that can be used as the underlying logic for theories whose truth conception is the quasi-truth – this logic is a kind of Jaśkowski discursive logic, a *paraconsistent modal logic*. We delineate some examples of applications in science and some philosophical considerations.

Key Words: Pragmatic truth, quasi-truth, partial structure, Kripke model semantics, paraconsistent modal logic, applications.

Introduction

The mathematical concept of pragmatic truth, first introduced by Mikenberg, da Costa and Chuaqui (1986) and subsequently named quasi-truth by da Costa, has received in the last few years several applications in logic and the philosophy of science.

The concept of quasi-truth is a generalization of Tarski's concept of truth. It is related to the concept of pragmatic truth according to pragmatist philosophers like James, Dewey and particularly Peirce, as well as to the idea of truth of thinkers like Vaihinger.

In this paper we discuss some connections between the concept of truth of these philosophers and the formal version of quasi-truth. By defining the mathematical concept of partial structure and by using a special semantic approach, we present a suitable logic that can be used as the underlying logic for theories whose truth conception is the quasi-truth.

We introduce two systems, so that the notions of pragmatic truth and pragmatic validity may be accommodated. We delineate a Kripke model semantics for this logic and among some fundamental results we show that there are important connections between this logic, modal logic and, in particular, Jaśkowski's discussive logic.

One of the main results is that the logic of pragmatic truth is paraconsistent and the philosophical import of this result, which justifies the application of pragmatic truth to inconsistent settings, will be also delineated.

1. The concept of truth and theories of truth

According, to da Costa:

> […] *we consider the classical concept of truth as a primitive concept. It is presupposed in all our practical and theoretical activities. Philosophically, truth is a final concept, indefinable through other simpler concept, if we use the term definition as a proposition that characterizes and explains, without petitio principii, a concept. The sentence itself expressing, in strict sense*, *the definition of truth would have to be "true"*. (da Costa (1999))

Lynch (1992) presents some connections between truth and other concepts: it is deeply connected to belief; it is also linked to knowledge; it is a central subject of logic in general; and it is also related to another mysterous concept, reality – in other words, to talk about truth is to talk of reality as it is.

We could investigate about two central subjects, concerning the *property* or *underlying nature* of truth:

1. Does truth even have a nature?
2. If so, what kind of nature?

The *deflationary theories* of truth answer the first question negatively.

The *robust theories* of truth try to answer the second question.

The *correspondence theories* of truth, the *coherence theories* and the *pragmatic theories* correspond to the robust theories of truth.

Correspondence truth is based on the idea that truth is correspondence with reality.

Generically, in a coherence theory of truth, a set of two or more beliefs is considered coherent if they "adjust" or "agree" among themselves.

Pragmatism, the philosophical movement founded by Charles Sanders Peirce, can be synthesized by the following passage, considered the "pragmatic maxim":

> *Consider what effects, that might conceivably have practical bearings, we conceive the object of our conception to have. Then, our conception of those effects is the whole of our conception of the object*. (Peirce (1931-1966) 5402)[1]

[1] References to Peirce's *Collected Papers* will be designated by [P], followed by the volume and paragraph numbers.

In this case, practical consequences are those effects of conception that have influence in our practice, in our action. The meaning of an idea consists of its practical effects on the human experience.

Pragmatism, then, is a theory of logical analysis, or true definition; and its merits are greatest in its application to the highest metaphysical conceptions. [[P] 6490]

Pragmatism can be understood as a method that tries to take the techniques of experimental investigation to the philosophical analysis.

A concept will be true if what it predicts will hold in the experience.

When Peirce speaks of pragmatic truth, he is making reference to the results of scientific investigation. (see D'Ottaviano and Hifume (2008))

2. Da Costa's quasi-truth

Alfred Tarski, in his famous paper of 1944, introduces a formal definition of truth, his *semantic conception of truth for formalized languages*, in order to capture the meaning of what he calls the Aristotelian classical conception of truth, a correspondence conception of truth.

Similarly, Mikenberg, da Costa and Chuaqui (1986) introduce a formal definition for the concept of pragmatic truth, later called quasi-truth by da Costa, trying to capture the meaning of the theories of truth of the pragmatist thinkers such as James, Dewey and particularly Peirce.

Loosely speaking, they say that a sentence is pragmatically true if, in a certain context, *"it saves the appearances*", i.e., if it is true in the classical correspondence sense.

According to Newton C.A. da Costa, we can establish a demarcation between philosophical interpretations of the notion of truth and a formal definition of truth.

The pragmatic, coherence and correspondence theories of truth are philosophical interpretations.

The pragmatist vision represents an emphasis in some considerations:

i) The nature of the agreement between imperfect or abstract descriptions and reality.
ii) The empirical consequences of such descriptions, understood as agreement with reality in the classic correspondence sense.
iii) The complete or absolute truth, again understood in the classic correspondence sense, as (ideal) end of every investigation.
iv) The representations are, basically, conceptually incomplete and unfinished and the adopted general attitude is fallibilistic.
v) Such representations, used in the scientific practice, are not seen as true in the correspondence sense, but as partially true, approximately true, or as containing some truth element.

From the naturalistic change in the philosophy of science, the nature and importance of scientific practice have been re-evaluated. A problem that appears is that no construction of reasoning can accommodate the vagueness and the complexities of such a practice.

The development of a formal definition of pragmatic truth or quasi-truth can eliminate the deficiencies of the attempts of formally capturing such notions.

Da Costa's definition of quasi-truth offers a way of accommodating the incompleteness inherent to scientific representations, by using a semantic approach. Tarski's definition of truth is extended by da Costa to the definition of quasi-truth.

The Tarskian notion of structure is extended, by introducing the notion of partial structure. And the notion of quasi-truth is proposed, being a generalization of Tarski's characterization of truth for partial contexts.

In general, when we investigate a certain domain of knowledge, we do not know everything about it, in others words, our information is partial or incomplete. We cannot surely claim that a particular theory concerning that domain is true; however, we can say, as much as our information allows us, that such theory can be true, that is, it is quasi-true.

When a determined domain Δ of knowledge is investigated, we submit it to a conceptual scheme, in order to systematize and to organize the information about it. This domain is "acted" by a set D of objects and is studied via the analysis of the relations among its elements.

Given a relation R, defined over D, as it is usual in the scientific contexts "we do not know" if all the objects of D (or n-tuples) are related by R.

On account of this, we say that our information concerning the domain of knowledge is incomplete or partial.

The introduction of the notions of partial relation and partial structure makes possible to formally accommodate that incompleteness and represent the information about the investigation domain.

Definition 2.1: Let D be a non-empty set. A n-ary *partial relation* R on D is a triple $\langle R_1, R_2, R_3\rangle$, where $R_i \cap R_j = \emptyset$, for $i \neq j$, $i, j \in \{1, 2, 3\}$ and $R_1 \cup R_2 \cup R_3 = D^n$ such that:

i) R_1 is the set of n-tuples that we know that belong to R;

ii) R_2 is the set of n-tuples that we know that do not belong to R;

iii) R_3 is the set of n-tuples that we do not know whether they belong to R or not.

We observe that if $R_3 = \emptyset$, R is an usual n-ary relation, that can be identified with R_1.

Definition 2.2: A *partial structure* is an ordered pair $\langle D, R_i\rangle_{i\in I}$, where:

i) D is a non-empty set;
ii) $(R_i)_{i\in I}$ is a family of i-ary partial relations on D.

As correspondence truth is involved in the notion of quasi-truth, also in the definition of quasi-truth Tarski's characterization of truth is involved.

For Tarski, a sentence of a first-order language L is true or false, only relatively to a certain interpretation in a given structure: similarly, a sentence can be quasi-true or quasi-false, only relatively to an appropriate type of structure.

As in Tarski's characterization only total structures are used (in which the relations are usual, non-partial), intermediate notions of structures are here defined, in order to establish a relationship between partial and total structures.

A simple pragmatic structure is a partial structure with a third component: a set P of sentences of L, either accepted as true or that are true according to the correspondence theory. These sentences can express either true statements, empirically decidable, or general sentences expressing either laws or theories accepted as true.

Definition 2.3: A *simple pragmatic structure* (*sps*) for a first-order language L is a structure $A = \langle D, R_k, P\rangle_{k\in I}$, where:

i) D is a non-empty set, the *universe* of A;
ii) R_k is a partial relation on D, for every $k\in I$ (R_k may be empty, for some k);
iii) P is a set of sentences of L.

Given a simple pragmatic structure, it can be extended to a total structure.

Definition 2.4: Let L be a first-order language, $A = \langle D, R_k, P\rangle_{k\in I}$ a *sps* and S a total structure, where L is interpreted. S is an *A-normal structure* if the following properties hold:

i) The universe *of* S is D;
ii) The total relations of S extend the correspondent partial relations of A;
iii) If c is an individual constant of L then c is interpreted in A and S by the same element;
iv) If $\alpha \in P$, then S satisfies α, i.e., every sentence of P is valid in the structure S, what is denoted by $S \models \alpha$.

Based on these notions, da Costa introduces his definition of a quasi-true, or pragmatically-true sentence.

Definition 2.5: Let L be a language, A a sps and S an A-normal structure. A sentence α of L is *quasi-true* in the sps A, relatively to S, if α is true in S according to Tarski's definition of truth. Otherwise, α is *quasi-false*.

A sentence can be either quasi-true or quasi-false only relatively to an appropriate type of structure – *the simple pragmatic structure.*

In other words, if α is quasi-true in A then all the logical consequences of α, or α plus the primary declarations P should be compatible with any primarily declaration.

Hence, α is such that everything happens in the domain of knowledge under investigation D as if α was true.

For da Costa, the concept of quasi-truth is the conception of truth inherent to the empirical theories.

3. A logic for quasi-truth

Now, we present a logical system, analysed in D'Ottaviano and Hifume (2007), that can serve as the underlying logic to theories that have the quasi-truth as their truth conception. This logic, a kind of Jaśkowski's discussive logic (see Jaśkowski (1969)), is a paraconsistent modal logic and can be, in general, used as a deductive logic of science.

In order to build this logic of pragmatic truth, from a first-order language *L* and a given sps *A* that interprets *L*, we consider its *A* - normal structures as "worlds" of a Kripke structure for the first-order modal system with equality $\mathbf{S_5Q^=}$, a $\mathbf{S_5Q^=}$ model. That is, from the universe of a sps *A* for *L*, we have several structures (total) in which *L* can be interpreted, such that any total structure is accessible to each other.

In *L* (and in *A*), the possibility operator $\Diamond$ corresponds to the quasi-truth notion (pragmatic truth) and the necessity operator ∇ to the quasi-validity notion (pragmatic validity). In order to formalize these two notions, we deal with two logical systems, $\mathbf{S_5Q^=}$ and **QT**.

The *pragmatically valid formulas* are the formulas α such that $\nabla\alpha$ is a theorem of $\mathbf{S_5Q^=}$. Among these, there are formulas $\nabla\Diamond\alpha$ such that $\Diamond\alpha$ is a theorem of $\mathbf{S_5Q^=}$.

We name the first class of formulas *strict-pragmatically valid*, or simply *strictly valid formulas* (the theorems of $\mathbf{S_5Q^=}$); the second class is named *pragmatically valid formulas*, that are the theorems of the system **QT**.

The language *L* of **QT** is the language of $\mathbf{S_5Q^=}$.

The axioms and inference rules are the following, where $\forall\forall\alpha$ is the closure of α.

Axiom 1. If a is an instance of a classical propositional tautology, then $\nabla\forall\forall\alpha$ is a **QT**-axiom.

Axiom 2. $\nabla\forall\forall(\nabla(\alpha \rightarrow \beta) \rightarrow (\nabla\ \alpha \rightarrow \nabla\ \beta))$.

Axiom 3. $\nabla\forall\forall(\nabla\ \alpha \rightarrow \alpha)$.

Axiom 4. $\nabla\forall\forall(\Diamond\ \alpha \rightarrow \nabla\Diamond\alpha)$.

Axiom 5. $\nabla\forall\forall(\forall\ x\ \alpha(x) \rightarrow \alpha(t))$.

Axiom 6. $\nabla\forall\forall(x = x)$.

Axiom 7. $\nabla\forall\forall(x = y \rightarrow (\alpha(x) \leftrightarrow \alpha(y)))$.

Axiom 8. In any formula, empty quantifications can be either introduced or suppressed.

Rule 1.

$$\frac{\vdash\nabla\forall\forall\alpha \qquad \vdash\nabla\forall\forall(\alpha \rightarrow \beta)}{\vdash\nabla\forall\forall\beta}$$

Rule 2. $\dfrac{\vdash \nabla\forall\forall\alpha}{\vdash\alpha}$

Rule 3. $\dfrac{\vdash \nabla\forall\forall\alpha}{\vdash\nabla\forall\forall\nabla\beta}$

Rule 4. $\dfrac{\vdash \nabla\forall\forall\alpha}{\vdash\alpha}$

Rule 5. $\dfrac{\vdash \nabla\forall\forall(\alpha\rightarrow\beta(x))}{\vdash\nabla\forall\forall(\alpha\rightarrow\forall \mathrm{x}\,\beta\,(x))}$

Hifume (2003) presents specific definitions and proves fundamental results of **QT**. (see also D'Ottaviano and Hifume (2007))

Definition 3.1: A ***QT**-model* is a $\mathbf{S_5Q^=}$ model.

Definition 3.2: In **QT**, a well formed formula (wff) α is a *semantic-pragmatic consequence of* a set Γ of wff of L, what is denoted by $\Gamma \models^{p}_{\mathbf{QT}} \alpha$, if, and only if, there are formulas $\delta_1, \delta_2, ..., \delta_n$ in Γ such that:

$$\models_{\mathbf{QT}} \Diamond\,\delta_1 \wedge \Diamond\delta_2 \wedge \ldots \wedge \Diamond\delta_n \rightarrow \Diamond\alpha.$$

Theorem 3.3: If a wff α is a theorem of $\mathbf{S_5Q^=}$, then it is a theorem of **QT**:

$$\vdash_{\mathbf{S_5Q^=}} \alpha \Rightarrow \vdash_{\mathbf{QT}} \alpha.$$

The converse of Theorem 3.3 is false, i.e., **QT** is "stronger" than $\mathbf{S_5Q^=}$.
We observe that the Barcan formula:

$$\forall x\nabla\alpha\,(x) \rightarrow \nabla\forall x\alpha\,(x).$$

holds in **QT**; and *Modus Ponens* does not hold, relatively to material implication.

Theorem 3.4: For every wff α of L, α is a theorem of **QT** if, and only if, $\Diamond\forall\forall\alpha$ is a theorem of $\mathbf{S_5Q^=}$:

$$\vdash_{\mathbf{QT}} \alpha \Leftrightarrow \vdash_{\mathbf{S_5Q^=}} \Diamond\forall\forall\alpha.$$

Definition 3.5: In **QT**, a formula α is a *syntactic-pragmatic consequence* of a set Γ of wffs, what is denoted by:

$$\Gamma \vdash^{p}_{\mathbf{QT}} \alpha,$$

if there are $\delta_1, \delta_2, ..., \delta_n$ in Γ such that:

$$(\Diamond\delta_1 \wedge \Diamond\delta_2 \wedge \ldots \wedge \Diamond\delta_n) \to \Diamond\alpha.$$

is a theorem in **QT**.

Definition 3.6: A *pragmatic theory* whose underlying logic is **QT**, is a non-empty set Σ of sentences such that, if $\delta_1, \delta_2, \ldots, \delta_n$ are in Σ and $\{\delta_1, \delta_2, ..., \delta_n\} \vdash^{p}_{\mathbf{QT}} \alpha$, then α is also in Σ.

Theorem 3.7: If Σ is a pragmatic theory and α is a theorem in **QT**, then $\alpha \in \Sigma$.

Definition 3.8: Let E be the set of all sentences of **QT** and Σ a pragmatic theory. Σ is *trivial*, or *overcomplete*, if $\Sigma = E$; otherwise, Σ is *non-trivial*. The theory Σ is *inconsistent (contradictory)*, if there exists at least a sentence α such that $\alpha \in S$ and $\neg\alpha \in S$, where $\neg$ is the negation symbol of **QT**; otherwise, Σ is *consistent (non-contradictory)*.

Theorem 3.9: There are inconsistent, but non-trivial, pragmatic theories.

Let us recall a definition of paraconsistent logics. (D'Ottaviano (1990))

Definition 3.10: A logic is *paraconsistent* if it can be used as the underlying logic for inconsistent but non-trivial theories, named *paraconsistent theories*.

In this sense, **QT** is a paraconsistent logic.

Let us introduce, by definition, new connectives in L, called *pragmatic connectives*.

Definition 3.11: Let α and β be wffs of L:

1. *Pragmatic implication* $\to_p$:
$\alpha \to_p \beta =_{\text{df}} \Diamond\, \alpha \to \beta$

2. *Pragmatic conjunction* $\wedge_p$:
$\alpha \wedge_p \beta =_{\text{df}} \Diamond\, \alpha \wedge \beta$

Let α, β be wffs of L. In general, the *Pseudo-Scotus Principle* does not hold in **QT**, relatively to the pragmatic implication, that is:

$$\nvdash_{\mathbf{QT}} \alpha \to_p (\neg\alpha \to_p \beta)$$

Hence, **QT** is *paraconsistent lato sensu*, or *non-explosive* and so paraconsistent, relatively to the pragmatic implication $\to_p$, according to other definitions of paraconsistent logic in the literature.

Proposition 3.12: For every wff α and β in **QT**, *Modus Ponens Rule* holds, relatively to the pragmatic implication, that is:

$$\text{If } \vdash_{\mathbf{QT}} \alpha \text{ and } \vdash_{\mathbf{QT}} \alpha \rightarrow_p \beta, \text{ then } \vdash_{\mathbf{QT}} \beta.$$

Following, we can prove the Pragmatic Completeness Theorem for **QT**.

Theorem 3.13 (Pragmatic Deduction Theorem): For every wff α and β, β is a syntactic-pragmatic consequence of α if, and only if, the pragmatic implication $\alpha \rightarrow_p \beta$ is a theorem in **QT**:

$$\alpha \vdash^{p}_{\mathbf{QT}} \beta \Leftrightarrow \vdash_{\mathbf{QT}} \alpha \rightarrow_p \beta.$$

Theorem 3.14 (Completeness Theorem): The wff α is pragmatically valid if, and only if, α is a theorem in **QT**:

$$\models_{\mathbf{QT}} \alpha \Leftrightarrow \vdash_{\mathbf{QT}} \alpha.$$

Theorem 3.15 (Pragmatic Completeness): Let Γ be a set of wff and α a wff of L. α is a semantical-pragmatic consequence of Γ if, and only if, α is a syntactic-pragmatic consequence of Γ in **QT**:

$$\Gamma \models^{p}_{\mathbf{QT}} \alpha \Leftrightarrow \Gamma \vdash^{p}_{\mathbf{QT}} \alpha.$$

4. Some examples of applications in empirical sciences

There are numerous situations, in the field of the empirical sciences, in which the concept of pragmatic truth can find applications.

1. Classical mechanics is at present known to be false. It was surmounted by relativistic mechanics. However, it can be applied in several domains, with appropriate limits. This occurs, for instance, in engineering, where nobody would suggest the use of relativity.

For the engineer, then, everything happens as though classical mechanics were true, i.e., as if it pictured reality. It is, for the engineer, quasi-true.

2. We may conclude, as a lesson of the history of science, that experience, in the wide acceptance of the word, will sooner or later refute any theory as an absolutely true picture of reality. Owing to this circunstance, as well as to many others, it is better not to envisage a theory as true but as quasi-true.

3. Sometimes we conceive theories simply as instruments to save the appearances or calculating devices in relation to observation sentences. In this case, we are actually saying that theories can be, at most, quasi-true.

5. A new version of the coherence theory of truth

Da Costa, Bueno and French (2005) have provided a new formulation of the coherence theory of truth using the resources of the partial structures approach – in particular the notions of partial structure and quasi-truth. They define when a sentence is *coherently true*.

The analogy between this characterization of truth as coherence and quasi (pragmatic)-truth is that the former is a syntactic version of the latter, and the latter is a semantic counterpart of the former.

6. Some philosophical considerations

A relevant book on science and partial truth was recently published by da Costa and French, in 2003.

An application of the quasi-truth theory in the foundations of the theory of probability and some extensions of it to inductive logic and the philosophy of science have been studied by da Costa and French (1993). This framework was also employed in order to examine some issues involved in the theory of acceptance, as well as in the modelling of "natural reasoning".

Quasi-truth can be also used to provide the epistemic framework for characterizing inconsistent belief systems.

Da Costa claims for the elaboration of an alternative account of belief, according to which "belief that *p*" is not to be understood as "belief that *p* is true", in the correspondence sense. He defends that, when it comes to representational structures such as scientific theories, "belief that *p*" is to be understood as "belief that *p* is pragmatically or partially true".

The occurence of inconsistent theories in science, for example, is now a widely recognized phenomenon; examples range from Bohr's theory of atom and "old" quantum theory of black-body radiation to the infinitesimal calculus and the Stone's analysis of the motion of a pendulum.

The problem is how to accommodate this aspect of scientific practice, given that within the framework of classical logic an inconsistent result is disastrous. This allows for the accommodation of inconsistency by acknowledging that it is not a permanent feature of reality to which theories must correspond, but is rather a temporary aspect of such theories which may nevertheless be extremely fruitful in a heuristic sense.

On this account it is not the logic of science, in the sense of the underlying logic of deduction and inference, which is paraconsistent, but rather the appropriate logic of truth.

Relatedly, the logic of quasi-truth may serve as a logic of scientific acceptance. The alternative proposed by da Costa is to retain the connection between belief and acceptance whilst rejecting truth-as-correspondence.

In this view, to accept a theory is to be committed, not to believing it to be true *per se*, but to holding it as if it were true, for the purposes of further elaboration, development and investigation.

This acceptance involves belief that the theory is partially true only and this, for da Costa, corresponds to the fallibilistic attitude of scientists themselves.

Linking acceptance and quasi-truth in that way restores a formal similarity between truth, taken generally, and acceptance with regard to deductive closure.

Within the framework of quasi-truth – recall the Jaśkowski discursive logic **QT** we can accommodate inconsistency, while still retaining a sense of deductive closure.

In this manner the relevance of logic to reasoning, especially scientific reasoning, is restored. (see da Costa, Bueno and French (1998))

We can also distinguish three inter-related areas in the philosophy of science to which the formalism of quasi-truth can be applied: the realistic-empiricist debate, the probalistic approach to confirmation and the problem of induction, and, finally, the nature and structure of scientific theories in general.

According to da Costa (see da Costa and French (2003)), possible future applications include theory evolution and inter-theoretical relations, the relationship between pragmatic truth and natural laws and the modelling of quantum mechanics in terms of simple pragmatic structures.

As da Costa's pragmatic theory encompasses the correspondence theory, we can say that the three major interpretations of the notion of truth – the correspondence, the coherence and the pragmatic accounts – can be put together in the formal framework we have delineated.

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